THE LOST NOTEBOOK AND PARTIAL THETA FUNCTIONS

S. OLE WARNAAR

In the Lost Notebook, Ramanujan wrote many amazing identities for partial theta functions, such as

$$1 + \frac{x^2(1-x)}{(1+ax^2)(1+\frac{x^2}{a})} + \frac{x^4(1-x)(1-x^3)}{(1+ax^2)(1+ax^4)(1+\frac{x^2}{a})(1+\frac{x^4}{a})} + \cdots$$

$$= (1+a)(1 - ax + a^2x^3 - a^3x^6 + \cdots) - a(1-x)(1-x^3)(1-x^5)\cdots \frac{1 - ax + a^3x^5 - a^6x^8 + \cdots}{(1+ax^2)(1+ax^4)\cdots(1+\frac{x^2}{a})(1+\frac{x^4}{a})\cdots}$$

and

$$1 + \frac{q(1-q)}{(1+aq)(1+\frac{q}{a})} + \frac{q^2(1-q)(1-q^3)}{(1+aq)(1+aq^2)(1+\frac{q}{a})(1+\frac{q^2}{a})}$$

$$= (1+a)(1 - aq + a^2q^3 - a^3q^6 + \cdots) - a\cdot\frac{(1-q)(1-q^3)\cdots(1 - a^2q^2 + a^5q^6 - \cdots)}{(1+aq)(1+aq^2)\cdots(1+\frac{q}{a})(1+\frac{q^2}{a})\cdots}$$

In this talk I will try to explain the origin of these identities, and will show that most partial-theta formulae from the Lost Notebook can be embedded in infinite hierarchies of such identities. This will reveal an unexpected connection between partial theta functions and Rogers–Ramanujan-type identities.

Department of Mathematics and Statistics, The University of Melbourne, Vic 3010, Australia

E-mail address: warnaar@ms.unimelb.edu.au